LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - **STATISTICS**

FIRST SEMESTER – **APRIL 2013**

ST 1815/1820 - ADVANCED DISTRIBUTION THEORY

Date : 27/04/2013 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

SECTION – A

Answer ALL the questions. (10 x 2 = 20 marks)

1. Let X_1 and X_2 have iid Poisson distribution. Obtain the conditional distribution of $X_1 \mid X_1 + X_2 = n$ at x.
2. Derive the MGF of a power series distribution.
3. Verify whether or not the exponential distribution satisfies lack of memory property.
4. Show that the truncated Poisson distribution, truncated at zero, is a power series distribution.
5. Prove that $\frac{1}{X}$ is distributed as Lognormal, if X is distributed as Lognormal.
6. If X is Inverse Gaussian, then prove that 2X is also Inverse Gaussian.
7. Establish that the marginals of a bivariate discrete uniform need not be discrete uniform.
8. Let $(X_1, X_2) \sim BVE(\lambda_1, \lambda_2, \lambda_{12})$. Obtain the distribution of $X_1 \wedge X_2$.
9. Define non-central chi-square distribution and write its mean.
10. Let $X \sim B(2, \theta)$, $\theta = 0.1, 0.2, 0.3$. If θ is discrete uniform, obtain the mean of the compound distribution.

SECTION - B

Answer any FIVE questions. (5 x 8 = 40 marks)

11. Let the distribution function of X be

$$F(x) = \begin{cases} 0, & x < -1 \\ \frac{(x+2)}{4}, & -1 \le x < 1 \\ 1, & 1 \le x < \infty \end{cases}$$

    Obtain (i) the decomposition of F and (ii) MGF of X.
12. State and prove a characterization result on Poisson distribution.
13. Establish that Binomial, Poisson and Log-Series distributions are Power-Series distributions.
14. Let $(X_1, X_2) \sim BB(n, p_1, p_2, p_{12})$. Stating the conditions, prove that (X_1, X_2) tends to $BVP(\lambda_1, \lambda_2, \lambda_{12})$.
15. Derive the regression equations associated with bivariate Poisson distribution.
16. Let (X_1, X_2) follow bivariate normal distribution with $V(X_1) = V(X_2)$. Check whether $X_1 + X_2$ and $(X_1 - X_2)^2$ are independent.
17. Obtain the mean and variance of non-central F distribution.
18. Given a random sample from a normal distribution, examine whether the sample mean is independent of the sample variance, using the theory of quadratic forms.

SECTION – C

Answer any TWO questions. (2 x 20 = 40 marks)

19(a) Let $X_1, X_2, \ldots, X_n$ be iid non-negative integer valued random variables. Prove that X_1 is geometric iff Min{ $X_1, X_2, \ldots, X_n$} is geometric. (10)

(b) State and prove the additive property of bivariate Poisson distribution. (10)

20(a) Obtain the cumulant generating function of power series distribution. Hence find the recurrence relation satisfied by the cumulants. (10)

(b) Let X_1 and X_2 be two independent normal variables with the same variance. State and prove a necessary and sufficient condition for two linear combinations of X_1 and X_2 to be independent. (10)

21(a) Define non-central t – statistic and obtain its pdf. (10)

(b) Let the distribution function of X be

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{(2x+1)}{4}, & 0 \leq x < 1 \\ 1, & x \geq 1 . \end{cases}$$

Obtain the mean, median and variance of X. (10)

22(a) Let $X_1 \sim G(\alpha, p_1)$, $X_2 \sim G(\alpha, p_2)$ and $X_1 \perp\!\!\!\perp X_2$. Then show that

(i) $X_1 + X_2 \sim G(\alpha, p_1 + p_2)$,

(ii) $X_1 \mid (X_1 + X_2) \sim$ Beta distribution of first kind and

(iii) $(X_1 + X_2) \perp\!\!\!\perp (X_1 \mid (X_1 + X_2))$. (16)

(b) Let X_1, X_2, X_3 be independent normal variables such that $E(X_1) = 1$, $E(X_2) = 3$, $E(X_3) = 2$ and $V(X_1) = 2$, $V(X_2) = 2$ and $V(X_3) = 3$. Examine the independence of $X_1 + X_2$ and $X_1 - X_2$. (4)

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